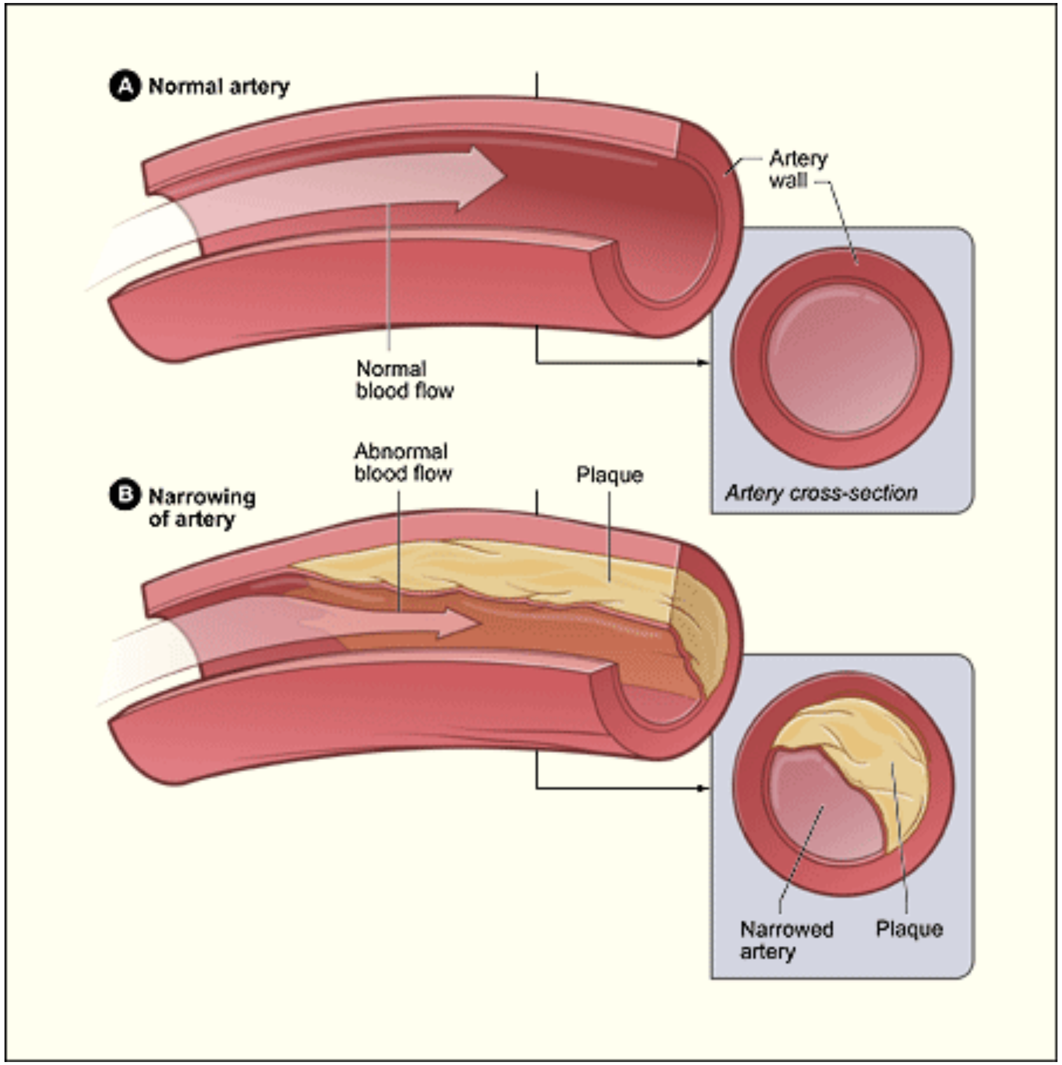
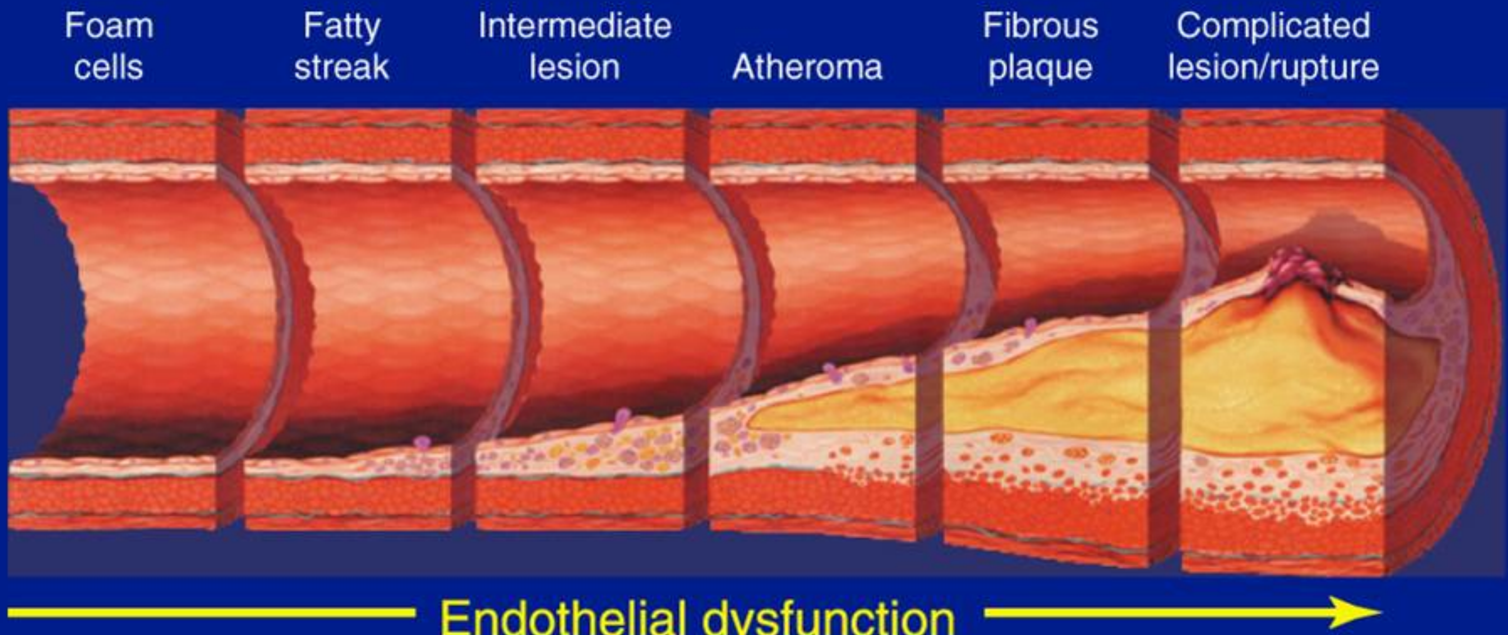
Arterial stenosis - atherosclerosis



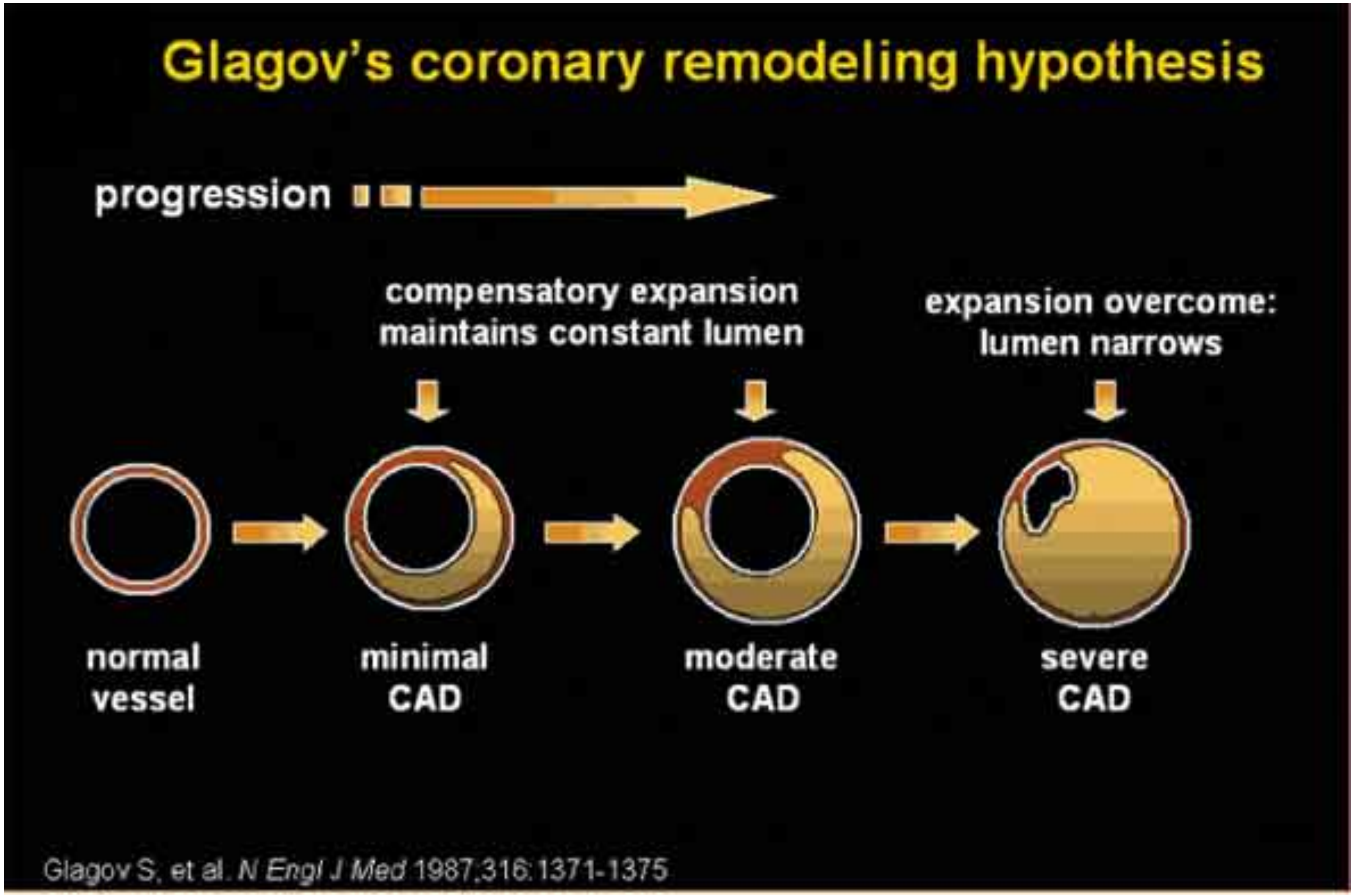
Atherosclerosis timeline



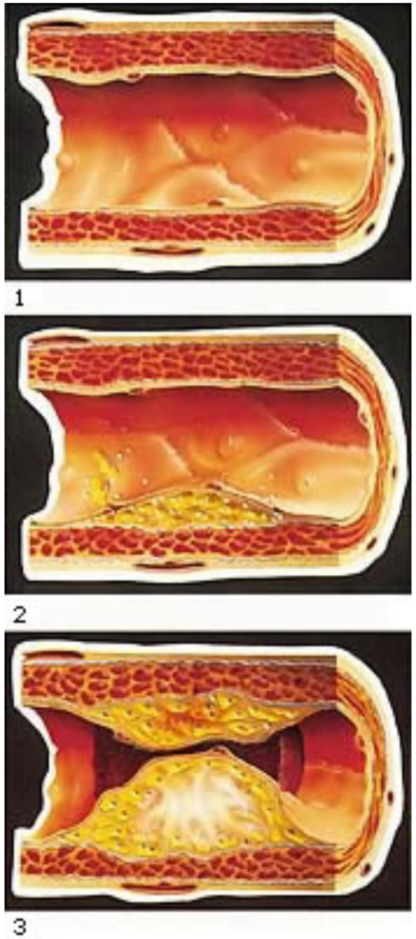
From first decade	From third decade	From fourth decade	
Growth mainly by lipid accumulation		Smooth muscle and collagen	Thrombosis, hematoma

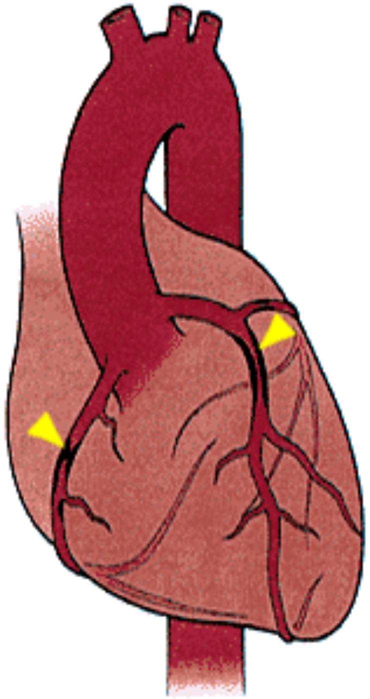
Pepine CJ. *Am J Cardiol.* 1998;82(suppl 10A):23S-27S.

Development of stenosis



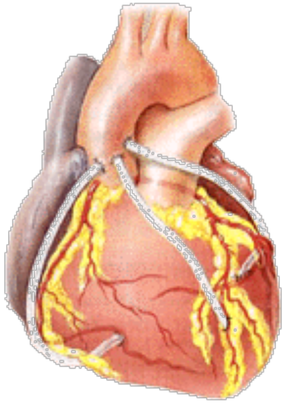
Atherosclerosis & coronary stenoses



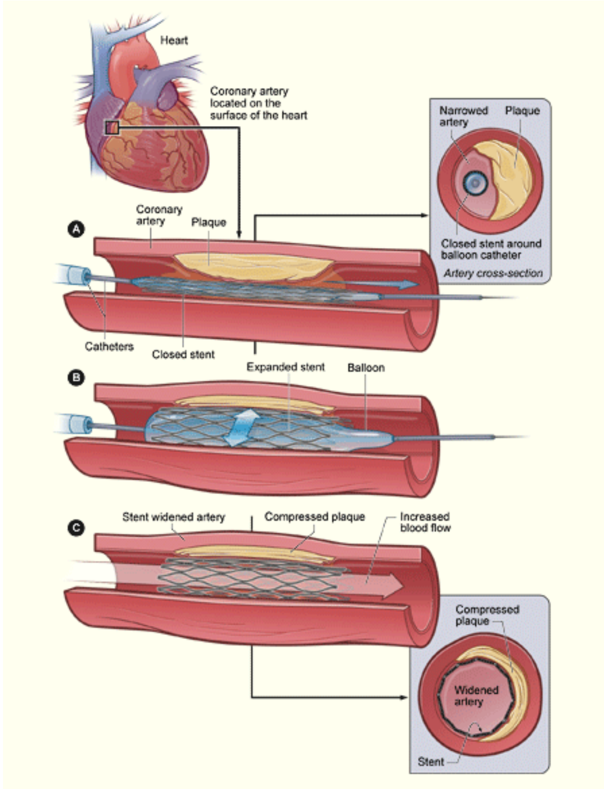


Treatment

Coronary bypass
(invasive)

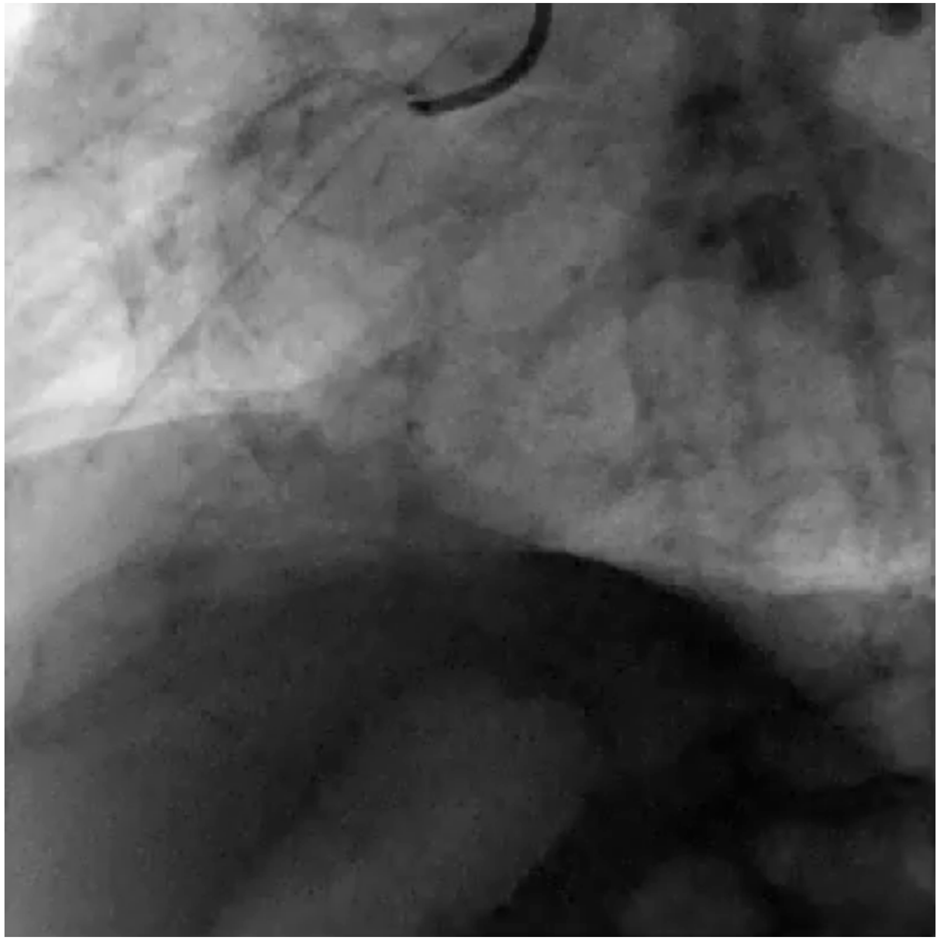


Coronary angioplasty
(minimally-invasive)

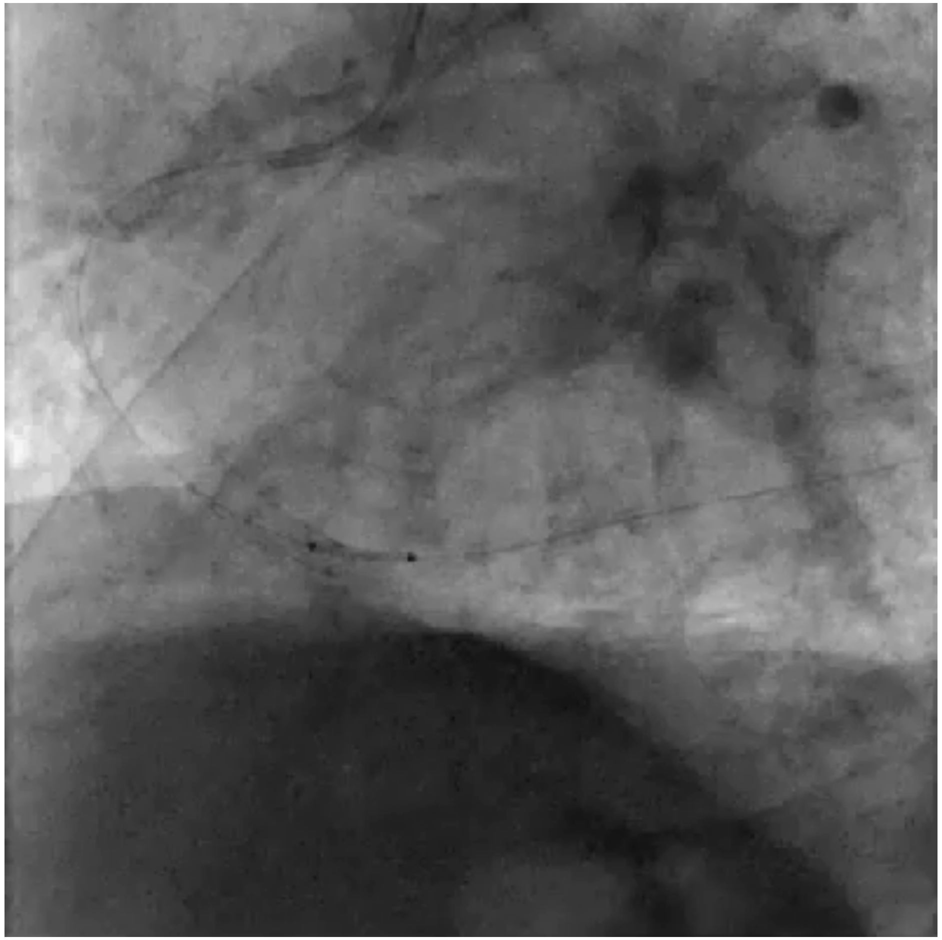


Angioplasty & stenting

Angioplasty & stenting (i)



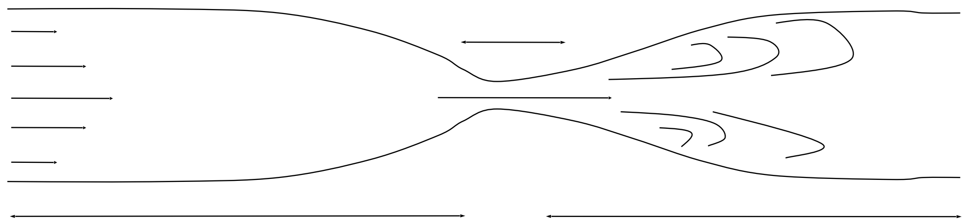
Angioplasty & stenting (ii)



Angioplasty & stenting (iii)

Arterial stenosis

Narrow section. If it is long and constant in diameter Poiseuille's law applies.

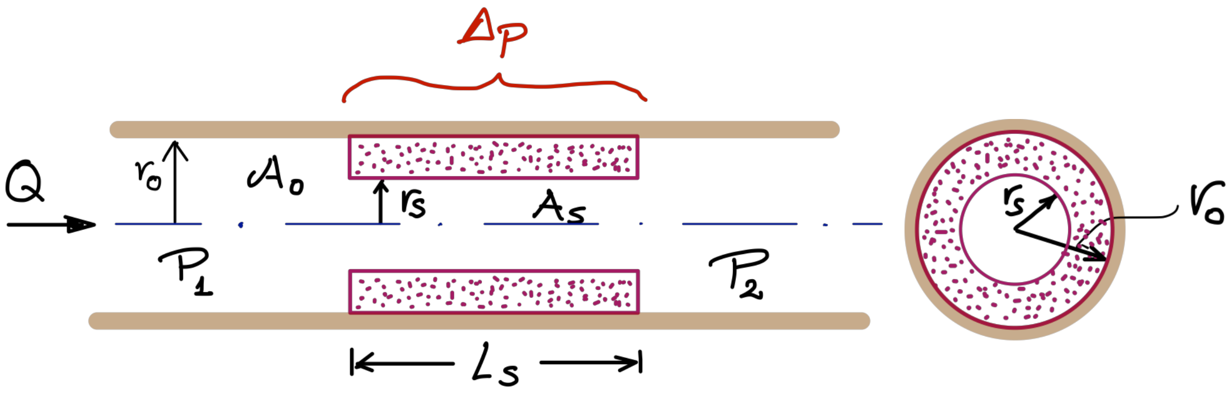


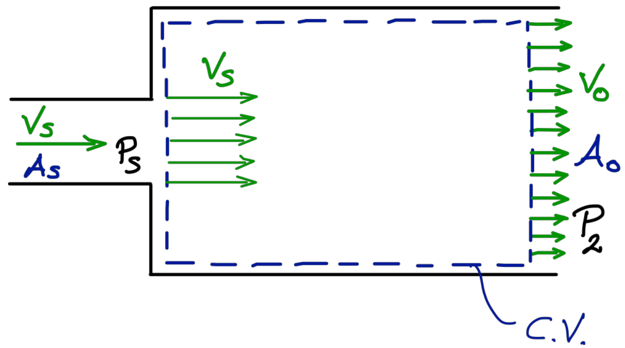
Converging tube: Zone where Bernoulli's equation applies.

Diverging tube: Flow separation & turbulence. Energy losses are important. Bernoulli's equation does not apply.

A COARCTATION consists of a converging section, a narrow section and a diverging section, each with their particular pressure-flow relations.

Pressure-flow relation across a stenosis





$$\Delta p = \Delta p_{viscous} + \Delta p_{turbulence} \qquad (1)$$

$$\Delta p_{viscous} = \frac{8 \mu L_s}{\pi r_s^4} Q \qquad (2)$$

$\Delta p_{turbulence}$: focus on the diverging flow field distal to the stenosis

Continuity: $V_s A_s = V_o A_o \qquad (3)$

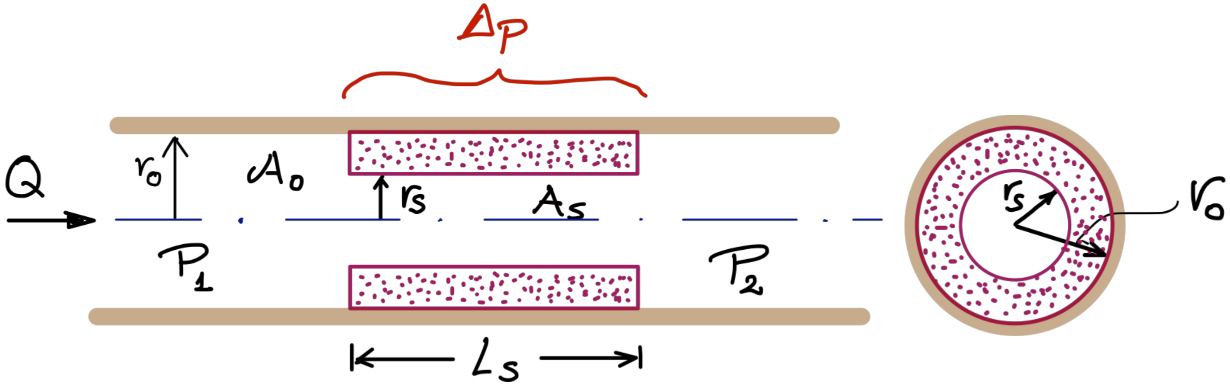
Momentum: $\Sigma F_x = \int_{cs} \rho \vec{V} \cdot \vec{V} \cdot \hat{n} \, dA$

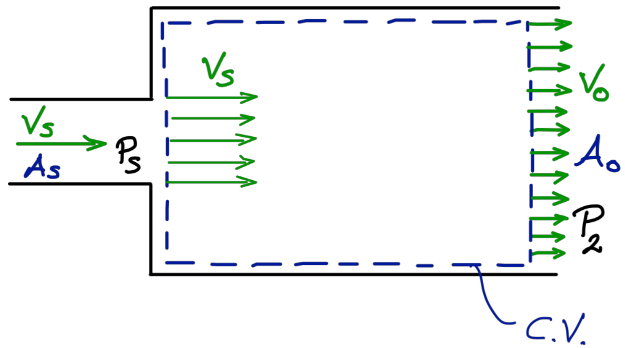
$$\Rightarrow P_s \cdot A_o - P_2 A_o = \rho V_s (-V_s) A_s + \rho V_o V_o A_o$$

$$= \rho V_o (V_o A_o) - \rho V_s (V_s A_s)$$

$$= \rho V_o A_o (V_o - V_s) \qquad (4)$$

Pressure-flow relation across a stenosis





Energy: $P_s + \frac{1}{2}\rho V_s^2 = P_2 + \frac{1}{2}\rho V_0^2 + \Delta P_{turbulence}$

$\Rightarrow \Delta P_{turb} = P_s - P_2 + \frac{1}{2}\rho\left(V_s^2 - V_0^2\right)$ (5)

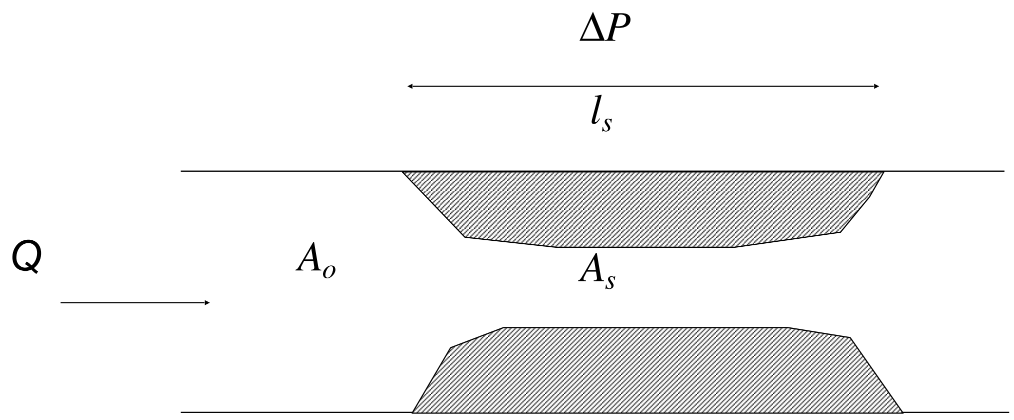
From (4) ∧ (5) $\Rightarrow \Delta P_{turb} = \rho V_0 (V_0 - V_s) + \frac{1}{2}\rho\left(V_s^2 - V_0^2\right) = \rho V_0^2 - \rho V_0 V_s + \frac{1}{2}\rho V_s^2 - \frac{1}{2}\rho V_0^2 = \frac{1}{2}\rho\left(V_0^2 - 2V_0V_s + V_s^2\right)$

$$= \frac{1}{2}\rho\left(V_0 - V_s\right)^2 = \frac{1}{2}\rho\left(\frac{Q}{A_0} - \frac{Q}{A_s}\right)^2 = \frac{1}{2}\rho\frac{Q^2}{A_0}\left[1 - \frac{A_0}{A_s}\right]^2$$

Hence, $\Delta P_{turb} = \frac{1}{2}\rho\frac{Q^2}{A_0}\left[\frac{A_0}{A_s} - 1\right]^2$ (6)

Finally, $\Delta P = \frac{8\mu L_s}{\pi r_s^4}Q + \frac{K_t\,\rho}{2A_0^2}\left[\frac{A_0}{A_s} - 1\right]^2 Q^2$ where K_t is an emperical coefficient, with $K_t \simeq 1.5$

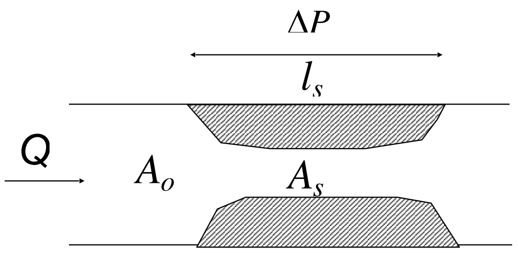
Pressure-flow relation across a stenosis



$$\Delta P = \frac{8\pi\mu l_s}{A_s^2} Q + \frac{K_t \rho}{2A_o^2}\left[\frac{A_o}{A_s} - 1\right]^2 Q^2$$

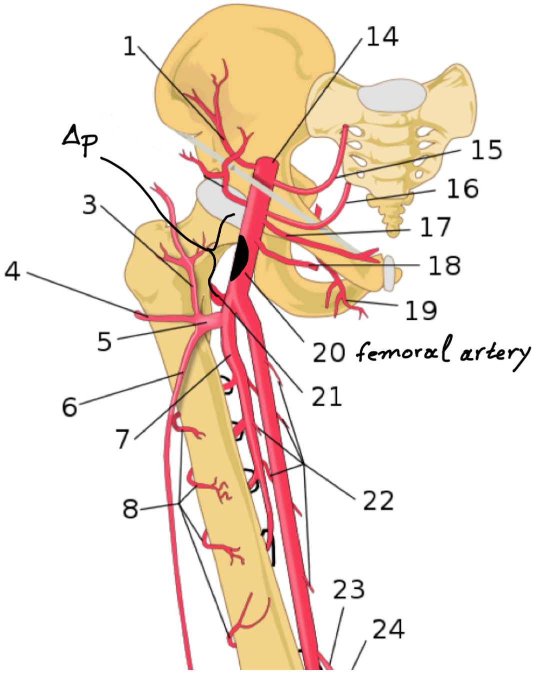
Viscous losses — turbulence

Pressure-flow relation across a stenosis: physiological and clinical significance
A: nonlinear effect of flow



$$\Delta P = \frac{8\pi\mu l_s}{A_s^2} Q + \frac{K_t \rho}{2A_o^2}\left[\frac{A_o}{A_s} - 1\right]^2 Q^2$$

Viscous losses — turbulence (dominant term)



$\Delta p \sim Q^2$

At normal, resting flow Q_n, the pressure drop across the stenosis is $\Delta p_n = 10$ mmHg.

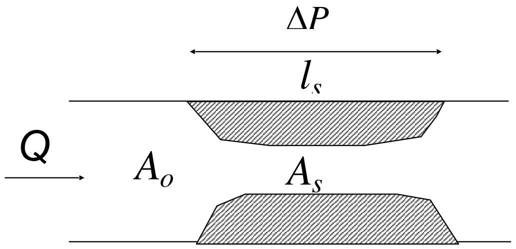
During exercise (fast walking), flow to the legs increases by a factor 3

$\Delta p_{ex} \sim Q_{ex}^2 = (3Q_n)^2 = 9\,Q_n^2$

$\Rightarrow \Delta p_{ex} = 9\,\Delta p_n = 90$ mmHg

Impossible! Flow limitation

Pressure-flow relation across a stenosis: physiological and clinical significance
B: nonlinear effect of area reduction (percent stenosis)



$$\Delta P = \underbrace{\frac{8\pi\mu l_s}{A_s^2} Q}_{\text{Viscous losses}} + \underbrace{\frac{K_t \rho}{2A_o^2}\left[\frac{A_o}{A_s} - 1\right]^2 Q^2}_{\text{turbulence (dominant term)}}$$

Definition: $\text{Percent stenosis} = \frac{A_o - A_s}{A_o} \times 100$

or $\%\ \text{stenosis} = \left(1 - \frac{A_s}{A_o}\right) \times 100$

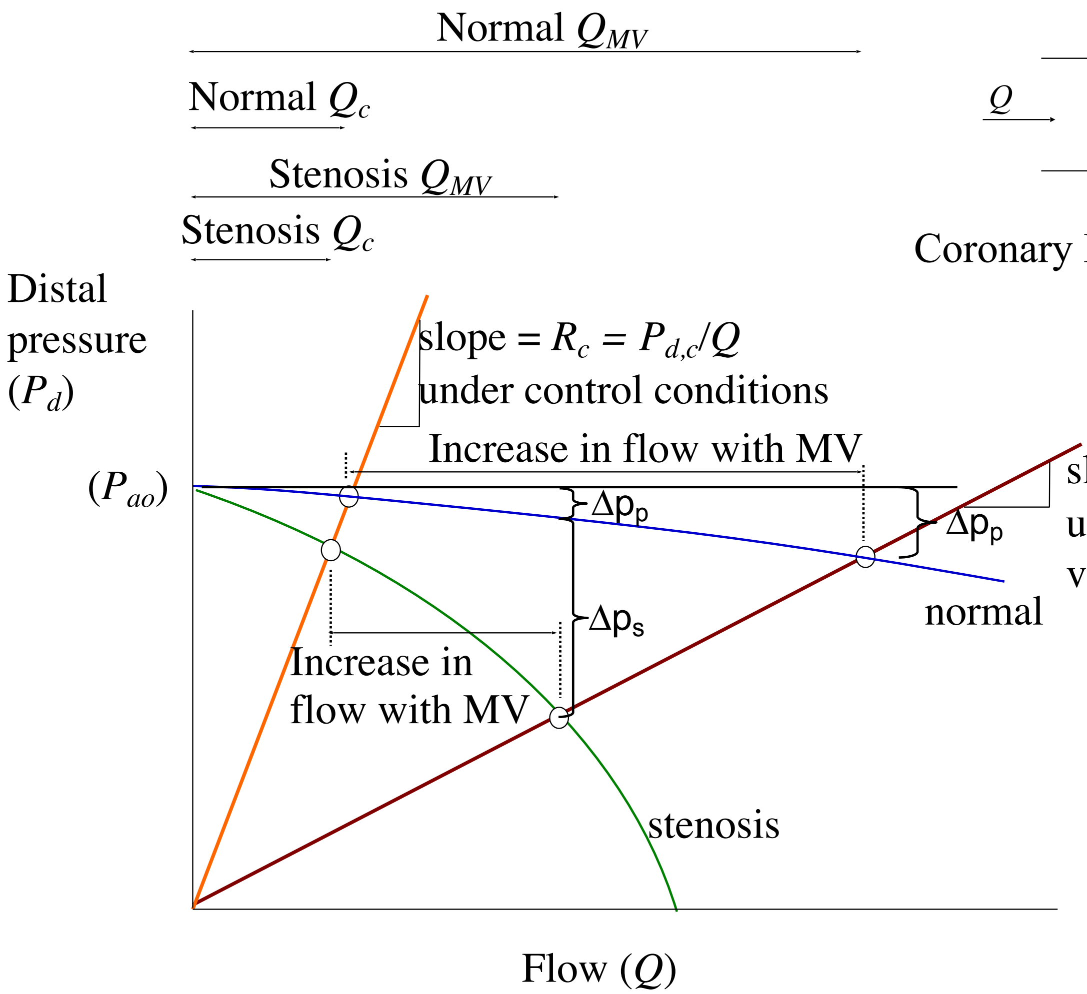
Example: An 80% coronary stenosis means that the free lumen within the stenosis, A_s, is only 20% of the arterial lumen, A_o

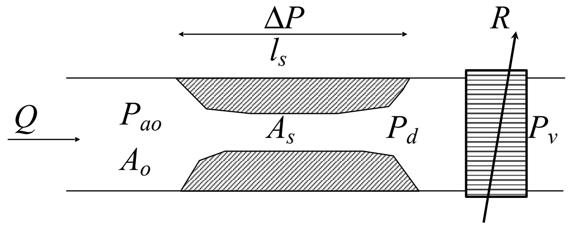
80% stenosis $\Rightarrow \frac{A_s}{A_o} = 20\%$

$$\Delta p \sim \left[\frac{A_o}{A_s} - 1\right]^2$$

% stenosis	$\left[\frac{A_o}{A_s} - 1\right]^2$
50%	$\left[\frac{1}{0.5} - 1\right]^2 = 1$
80%	$\left[\frac{1}{0.2} - 1\right]^2 = 4^2 = 16$
90%	$\left[\frac{1}{0.1} - 1\right]^2 = 9^2 = 81$

Example: coronary stenosis

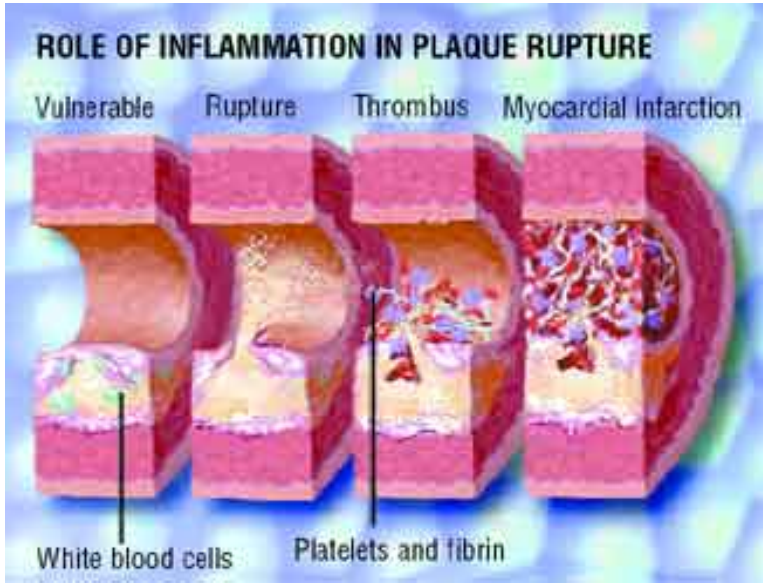


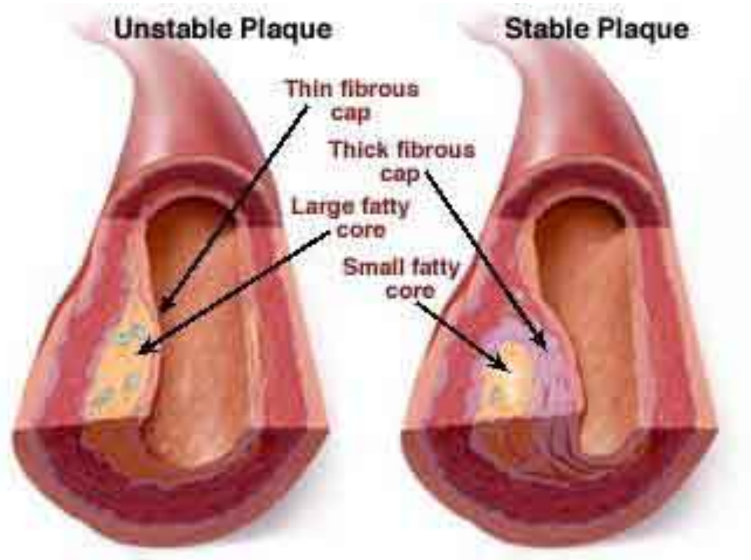


Coronary Flow reserve = Q_{MV}/Q_c

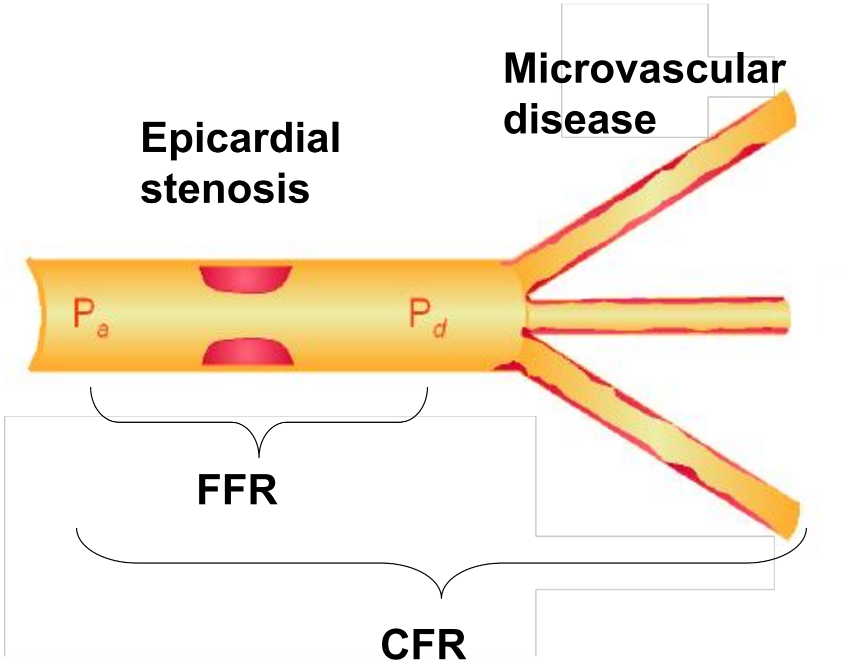
MV: maximal vasodilation

Stable vs. unstable stenosis





Percutaneous coronary intervention / model development



$$CFR = \frac{Q_{MV}}{Q_n}$$

Coronary Flow Reserve (CFR)
(hyperemic/baseline)

$$FFR = \frac{P_d}{P_a}$$

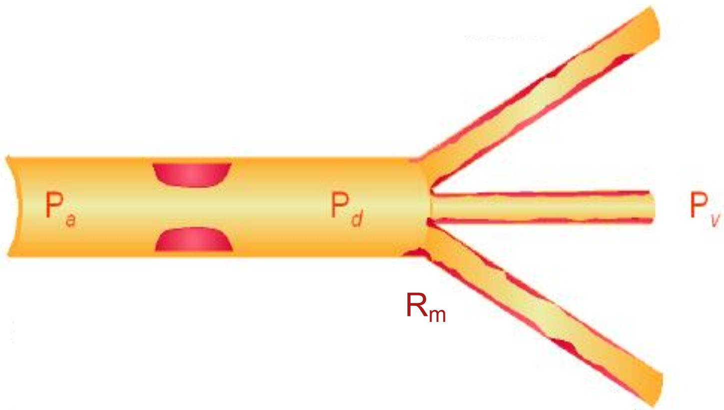
Functional Flow Reserve
(FFR)
(stenosed/normal)

CFR → coronary flow measurements (difficult)

FFR → coronary pressure measurements (easier, guide wire)

FFR → coronary pressure measurements (easier, guide wire)

Assumptions:
- $P_v << P_a$
- Arterial pressure remains the same in presence or absence of stenosis
- Distal resistance remains the same in presence or absence of stenosis



$$FFR_Q = \frac{Q_s}{Q_n} = \frac{(P_{d,s} - P_v)/R_m}{(P_{a,n} - P_v)/R_m} \approx \frac{P_{d,s}}{P_{a,s}} = FFR_P$$

n: normal, absence of stenosis
s: in presence of stenosis

Usually FFR < 0.8 => indication to intervene with stenting

Pijls, et al., Circulation, (2000)